

- ① Solve the differential equation. Show every step clearly.

$$\frac{dy}{dx} = \frac{e^x \sin(3x)}{y}$$

- ② You are standing at the top of a 2000 meter tall building. A bucket containing 10 kg of water is hanging on the end of a uniform 90 meter chain from the top of the building. The chain weighs a total of 100 kg. How much work is required for you to pull the bucket and chain to the top of the building? Make sure you justify each step.

- ③ Calculate the integral. Show each step

$$\int_{1/15}^{\sqrt{3}/15} \frac{x^6}{\sqrt{4-25x^2}} dx$$

- ④ Calculate the Taylor series for $f(x) = x \cos x$ for x near 0. Calculate the first four terms directly. Write out the entire series using summation notation.

- ⑤ Solve the differential equation. Provide clear detailed and complete solution.

$$\frac{dy}{dx} = \frac{x(y^2 - 1)}{2x^2 - 6x + 4}$$

- ⑥ Determine if the following series is absolutely convergent, conditionally convergent, or divergent. Show a clear justification for each part of your solution

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$$

- ⑦ Consider the parametrized curve by:
 $x = e^{2t} \sin t$ and $y = \cos(4t)$

find the slope-intercept equation for the tangent line to this curve when $t=1$. Show every step

- ⑧ Consider the region bound by the curves $y=x$, $y = \frac{9x}{1+x^3}$. Revolve this region about the $x=1$ super fast so that it looks like a solid object. Find the volume of the solid. Show every step.

⑨ Consider the integral:

$$\int_0^{1/2} \frac{1}{1+x^{10}} dx$$

Use the power series to approximate the integrand $\frac{1}{1+x^{10}}$ and calculate the integral. Show every step.